Active control of ionized boundary layers

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Abstract

The challenging problems, in the field of control of chaos or of transition to chaos, lie in the domain of infinite-dimensional systems. Access to all variables being impossible in this case and the controlling action being limited to a few collective variables, it will not in general be possible to drive the whole system to the desired behaviour. A paradigmatic problem of this type is the control of the transition to turbulence in the boundary layer of fluid motion. By analysing a boundary layer flow for an ionized fluid near an airfoil, one concludes that active control of the transition amounts to the resolution of an generalized integro-differential eigenvalue problem. To cope with the required response times and phase accuracy, electromagnetic control, whenever possible, seems more appropriate than mechanical control by microactuators.

1 Introduction

Control of chaos or of the transition to chaos has been, in recent years, a very active field (see for example Ref.[1] and references therein). Several techniques were developed and tested, mostly for low dimensional dynamical systems. The challenge lies now on finding out whether these techniques extend to infinite-dimensional systems.

A first aspect preventing a simple extrapolation of the finite-dimensional techniques is the fact that only a small subset of variables (or some integrated collective variable) is acessible to measurement. Likewise the variables on which one may act for controlling purposes are even more limited. A second aspect is that, rather than to stabilize an unstable periodic orbit (a single mode), what one aims in general is to suppress a continuous set of unstable modes, or to stabilize a particular collective mode and, at the same time, prevent all other modes from developing. In this sense the problem is no longer a standard control problem to be handled by pole placement, sliding mode or other standard technique. Instead, as suggested by the problem discussed in this paper the control problem amounts to the solution of a generalized integro-differential eigenvalue problem.

A problem of both theoretical and practical importance is the control of the transition from laminar to turbulent motion in a boundary layer flow. I deal with this problem mostly as an example and prototype of the kind of questions and mathematical framework to be expected in the control of chaos for infinite-dimensional complex systems. However for the benefit of the reader less familiar with aerodynamical issues I have included a few remarks on the physical and technological context of the problem.

By delaying the laminar to turbulent transition, an order of magnitude reduction in the skin friction drag is achieved. The technological benefits that may be derived from this reduction, led to the proposal of several methods for the control of the boundary layer transition. They are both of passive and active type and include pressure gradient control, wall suction, wall temperature control, polymer coating, compliant walls, etc.

In passive type control[2] [3], the aim is either to induce a modification of the curvature of the velocity profile, or to break the eddies and absorb their energy.

On the other hand the active control methods, that have been proposed[4], aim at cancelling the growth of the Tollmien-Schlichting (TS) waves, a known precursor of the transition instability. This is achieved by creating a disturbance of opposite phase to cancel the TS waves. The wave cancelling disturbance may be created, for example, by modulated suction and blowing or by mechanical microactuators. This control requires an accurate set of sensors and actuators. The reaction time of the actuators is critical to achieve control, especially if one aims at the feedback cancellation of nonlinear effects. The fact that some of the spatial growing modes have high frequencies, leads

to the suspicion that mechanical sensors and actuators, even if highly miniaturized, will have an hard time to deal with the high frequency instabilities that are known to be present in the transition.

Greater speed and flexibility would be achieved were it possible to act on the flow by electromagnetic fields. With the possible exception of electrolytes like seawater, a direct electromagnetic action on the unmodified fluid[7] [8] [9] [10] [11] [12] [13] does not seem possible. However, even for neutral fluids, improved control of the boundary layer flow might be achieved by injecting in the leading edge of the airfoil a stream of ionized gas, creating a thin ionized layer which might then be acted upon by electromagnetic fields. In Ref.[3] a detailed discussion is carried out of the effect of a streamwise directed electric field on the velocity profile of an ionized boundary layer, taking into consideration the fact that an injected stream of ionized gas leads to a nonuniform charge profile. The study establishes reference values and design estimates for the electric fields and ionization densities required for a significant change of the velocity profile.

In the present paper a methodology is studied to assess the possibilities of electromagnetic control of the TS precursor waves. Usually one thinks of active control in terms of laminarizing the boundary layer flow. However the opposite situation may also occur because, for example, in stalling prone situations it might be useful to induce turbulence to avoid separation. Then the fast reaction time of electromagnetic control might also be an asset.

2 The stability equations

Consider the Navier-Stokes equation

$$\frac{\partial \widetilde{U}}{\partial t} + (\widetilde{U}.\nabla)\widetilde{U} = -\frac{1}{\widetilde{\rho}_m}\nabla \widetilde{p} + \widetilde{\nu}\triangle \widetilde{U} + \frac{\widetilde{\sigma}}{\widetilde{\rho}_m}\widetilde{E} + \frac{\widetilde{\sigma}}{c\widetilde{\rho}_m}\widetilde{U}\times \widetilde{B} \tag{1}$$

for an incompressible ionized fluid in an external electromagnetic field

$$\frac{\partial \widetilde{\rho}_m}{\partial t} + \nabla \cdot \left(\widetilde{\rho}_m \widetilde{U}\right) = 0 \tag{2}$$

In orthogonal curvilinear coordinates, denote by $(\widetilde{u}, \widetilde{v}, \widetilde{w})$ the streamwise, the normal and the spanwise components of the physical velocity field $\widetilde{U}$. Define

also reference quantities and adimensional variables

$$\begin{array}{l} x=\frac{\widetilde{x}}{L_r}; \quad y=\frac{\widetilde{y}}{\delta_r}; \quad t=\widetilde{t}\frac{U_r}{L_r}; \quad u=\frac{\widetilde{u}}{U_r}; \quad v=\frac{\widetilde{v}L_r}{U_r\delta_r}; \quad w=\frac{\widetilde{w}}{U_r} \\ \rho_m=\frac{\widetilde{\rho_m}}{\rho_r}; \quad p=\frac{\widetilde{p}}{\rho_r U_r^2}; \quad \nu=\frac{\widetilde{\nu}}{\nu_r}; \quad \sigma=\frac{\widetilde{\sigma}}{\sigma_r}; \quad E=\frac{\widetilde{E}}{E_r} \end{array} \tag{3}$$

Typical values for the reference quantities, as used before[3], are $U_r = 100$ m s^{-1}, $L_r = 1$ m, $\delta_r = 10^{-3}$ m, $\rho_r = 1.2$ Kg m^{-3}, $E_r = 500$ V cm^{-1}, $\sigma_r = 15$ μC cm^{-3}, $\nu_r = 1.5 \times 10^{-5}$ m^2 s^{-1}. Then $\mathrm{R}_L = \frac{U_r L_r}{\nu_r} = 6.66 \times 10^6$ and $\frac{1}{R_L}$ and $\frac{\delta_r^2}{L_r^2} = 10^{-6}$ are small quantities.

Expressing (1) in the adimensional variables (3), assuming that the product $k\delta$ of the airfoil curvature times the boundary layer width is small and neglecting terms of order R_L^{-1}, $\frac{\delta_r^2}{L_r^2}$ and $\frac{\widetilde{U}}{c}$ one is left with

$$\begin{array}{l} \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho_m}\frac{\partial p}{\partial x}+\nu\omega\frac{\partial^2 u}{\partial y^2}+\frac{\gamma}{\rho_m}\sigma E_x \\ \frac{\partial p}{\partial y}=\frac{\delta_r}{L_r}\gamma\sigma E_y \\ \frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho_m}\frac{\partial p}{\partial z}+\nu\omega\frac{\partial^2 w}{\partial y^2}+\frac{\gamma}{\rho_m}\sigma E_z \end{array} \tag{4}$$

where $\gamma = \frac{L_r \sigma_r E_r}{U_r^2 \rho_r}$ and $\omega = \frac{L_r^2}{\delta_r^2 R_L}$ ($\gamma = 62.5$ and $\omega = 0.15$ for the reference values above)

The aim is to study the stability of the steady state (laminar) solutions of the above equations with regard to the precursor waves that develop in the transition region. Therefore the variables are decomposed into steady state ($\overline{u}, ...$) and fluctuating components ($u^{'}, ...$)

$$\begin{array}{l} u=\overline{u}+u^{'} \\ v=\overline{v}+v^{'} \\ w=\overline{w}+w^{'} \\ p=\overline{p}+p^{'} \\ E=\overline{E}+E^{'} \end{array} \tag{5}$$

and one looks for normal mode solutions of the form

$$\left\{\begin{array}{c} u^{'} \\ v^{'} \\ w^{'} \\ p^{'} \end{array}\right\}=\left\{\begin{array}{c} \widehat{u}(y) \\ \widehat{v}(y) \\ \widehat{w}(y) \\ \widehat{p}(y) \end{array}\right\}\exp\left\{i\left(\alpha x+\beta z-\Omega t\right)\right\} \tag{6}$$

with a similar, but $y-$independent, form for the electric field

$$E^{'} = \widehat{E} \exp \left\{ i \left(\alpha x + \beta z - \Omega t \right) \right\} \tag{7}$$

From the control point of view this implies the capability to have the electric field react to the fluctuating velocity field with the same frequency and wavelength, but eventually with some delay represented by the phase of the complex amplitude $\widehat{E}$. To have this feedback response, a distributed set of sensors should be available on the surface of the airfoil. The sensors, of course, cannot measure the velocity field itself but only some integrated effect, observable at the coordinate $y = 0$ (see below).

In the transition region the quasiparallel hypothesis for the stationary solution is a good approximation. Namely $\overline{v} = \overline{w} = \frac{\partial \overline{u}}{\partial x} = 0$. This holds for example for one of the scaling solutions in Ref.[3]

$$\overline{u} = u_e \left(1 - \exp \left(-y \frac{\chi}{\sqrt{u_e}} \right) \right) \tag{8}$$

for $\chi = \sqrt{\frac{\gamma \sigma_0 \overline{E}_x}{\omega \nu \rho_m}}$ and a charge distribution profile

$$\sigma = \sigma_0 \left(1 - \frac{\overline{u}}{u_e} \right) \tag{9}$$

It is the stability and controllability of this solution that is going to be studied.

By differentiating Eqs.(4) the pressure terms may be eliminated. Then, keeping only the linear terms in the fluctuating fields and using (6) and (7), one obtains

$$\begin{aligned}
\nu \omega \widehat{v}^{''''} + i \theta \widehat{v}^{''} &= i \alpha \left(\overline{u}^{'} + \overline{u}^{''} \right) \widehat{v} + i \alpha \left(\overline{u} + \overline{u}^{'} \right) \widehat{v}^{'} \\
&\quad + i \tfrac{\gamma}{\rho_m} \left\{ \sigma^{'} \left(\alpha \widehat{E}_x + \beta \widehat{E}_z \right) - i \tfrac{\delta_r}{L_r} \sigma \left(\alpha^2 + \beta^2 \right) \widehat{E}_y \right\} \\
\nu \omega \widehat{w}^{'''} + i \theta \widehat{w}^{'} &= i \alpha \overline{u}^{'} \widehat{w} + i \alpha \overline{u} \widehat{w}^{'} - \tfrac{\gamma}{\rho_m} \left\{ \sigma^{'} \widehat{E}_z - i \beta \tfrac{\delta_r}{L_r} \sigma \widehat{E}_y \right\}
\end{aligned} \tag{10}$$

with the boundary conditions

$$\widehat{v}(0) = \widehat{v}(\infty) = \widehat{w}(0) = \widehat{w}(\infty) = \widehat{v}^{'}(0) = \widehat{v}^{'}(\infty) = \widehat{w}^{''}(\infty) = 0 \tag{11}$$

The last three boundary conditions are obtained from the continuity equation

$$i\alpha\widehat{u} + \widehat{v}^{'} + i\beta\widehat{w} = 0 \tag{12}$$

and the last equation in (4).

In situations where the spanwise fluctuations may be neglected, the flow becomes two-dimensional and a stream function may be defined for the waves

$$\begin{array}{rcl} u^{'} = \frac{\partial\psi}{\partial y} & ; & v^{'} = -\frac{\partial\psi}{\partial x} \\ \psi & = & \phi(y)\exp\left\{i\left(\alpha x + \beta z - \Omega t\right)\right\} \end{array} \tag{13}$$

Then the stability equation is

$$\nu\omega\phi^{''''} + i\theta\phi^{''} \quad = \quad i\alpha\left(\overline{u}\phi^{''} + \overline{u}^{''}\phi\right) - \frac{\gamma}{\rho_m}\left\{\sigma^{'}\widehat{E}_x - i\alpha\frac{\delta_r}{L_r}\sigma\widehat{E}_y\right\} \tag{14}$$

which is a simplified version of the Orr-Sommerfeld equation with a driving term. The simplification arises from the fact that terms of order $1/R_L$ and δ_r^2/L_r^2 have been neglected. In this form the equation may be integrated once and reduced to a third order problem (see below).

3 Stability and controllability results

Consider first the spanwise stability of the scaling solution (8) without control ($\widehat{E}_z = \widehat{E}_y = 0$). The second equation in (10) may be integrated once and the integration constant set to zero using the boundary conditions (11). Using the scaling solution (8) for $\overline{u}$ and changing coordinates to

$$\eta = 1 - \exp\left(-y\frac{\chi}{\sqrt{u_e}}\right) \tag{15}$$

one obtains

$$\left\{(1-\eta)^2\frac{d^2}{d\eta^2} - (1-\eta)\frac{d}{d\eta}\right\}\widehat{w} + i\theta_1\widehat{w} = i\alpha_1\eta\widehat{w} \tag{16}$$

where $\theta_1 = \frac{\theta u_e}{\nu\omega\chi^2}$ and $\alpha_1 = \frac{\alpha u_e^2}{\nu\omega\chi^2}$, with boundary conditions

$$\widehat{w}(0) = \widehat{w}(1) = 0 \tag{17}$$

Discretizing the $(0,1)$ interval, the calculation of the largest growing modes becomes an algebraic generalized eigenvalue problem which is dealt with by the QZ algorithm. In Fig.1a one plots the largest value of $\mathrm{Re}(i\alpha_1)$ for real θ_1 and in Fig.1b the largest value of $\mathrm{Re}(-i\theta_1)$ for real α_1. All modes having negative real parts, the conclusion is that the scaling solution is both space- and time- spanwise stable. Therefore we may take $\widehat{w}=0$ and use, for the streamwise stability, the stream function stability equation (14).

For the scaling solution (8), with the same change of variables, neglecting the term in $\widehat{E}_y$ because $\frac{\delta_r}{L_r}$ is a small quantity, integrating once the equation and fixing the integration constant with the boundary conditions, the result is the equation

$$\begin{aligned}&\left\{(1-\eta)^3\tfrac{d^3}{d\eta^3}-3(1-\eta)^2\tfrac{d^2}{d\eta^2}+(1-\eta)\tfrac{d}{d\eta}\right\}\phi+i\theta_1(1-\eta)\tfrac{d\phi}{d\eta}\\&\quad=i\alpha_1\left\{\eta(1-\eta)\tfrac{d}{d\eta}-(1-\eta)\right\}\phi-\tfrac{\gamma\sigma_0 u_e^{3/2}}{\rho_m\nu\omega\chi^3}(1-\eta)\widehat{E}_x\end{aligned} \tag{18}$$

with boundary conditions

$$\phi(0)=\phi^{'}(0)=\phi(1)=0 \tag{19}$$

Let first $\widehat{E}_x=0$ (uncontrolled equation). Using as before a finite difference method and the QZ algorithm with the boundary conditions imposed as three of the equations in the algebraic system, one obtains, for the largest value of $\mathrm{Re}(i\alpha_1)$ and real θ_1, the results shown in Fig.2. It means that there is a range of frequencies for which there is spatial growth of the streamwise fluctuations. Therefore the scaling solution is spatially unstable.

To derive a controlled equation one has to realize that the only physical quantities, that it is reasonable to assume to be observable, are the pressure fluctuations on the airfoil or the integrated effect of the electrical current as seen at the surface of the airfoil. Pressure fluctuations may be detected by a distributed set of microphones and the integrated electrical current fluctuations are essentially the induced magnetic field fluctuations on the spanwise direction. For definiteness I will assume that a set of sensors is available to measure the effect of the electrical current fluctuations. To achieve control this measurement is used to modulate a variable component of the applied electric field. That is

$$\widehat{E}_x=k\int_0^\infty dy\sigma_0\left(1-\frac{\overline{u}(y)}{u_e}\right)\widehat{u}(y) \tag{20}$$

with k a complex proportionality constant, the meaning of the phase being the control delay. Then, the controlled equation is

$$\begin{array}{c} \left\{(1-\eta)^3\frac{d^3}{d\eta^3} - 3(1-\eta)^2\frac{d^2}{d\eta^2} + (1-\eta)\frac{d}{d\eta}\right\}\phi + i\theta_1(1-\eta)\frac{d\phi}{d\eta} \\ = i\alpha_1\left\{\eta(1-\eta)\frac{d}{d\eta} - (1-\eta)\right\}\phi - C(1-\eta)\int_0^1 d\eta\phi \end{array} \tag{21}$$

with $C = \frac{k\gamma\sigma_0^2 u_e^{3/2}}{\rho_m \nu\omega\chi^3}$. The controllability problem amounts to find out whether all eigenvalues have negative real parts in the integro-differential problem defined by Eq.(21). Let $\theta_1 = 60$, the frequency for which the largest $\mathrm{Re}(i\alpha_1)$ is at its maximum. For real C, Fig.3 shows that for $C > 1$ the largest mode has spatial decay, hence the solution becomes stable. For the results in Fig.4, let $C = |C|\, e^{i\varphi}$ with $|C| = 1.5$ and variable phase φ. One sees that there is a range of phase delays which stabilize the solution and, conversely, outside this range the solution is strongly unstable.

4 Conclusions

1. Stabilization of a stationary configuration in an infinite-dimensional system involves the study of infinitely many disturbance modes, some of which may grow in time and space. In addition, the measurable observables, to which some local control may react, involve the integrated effect of many variables. Therefore the mathematical structure of the problem to be solved is expected to be, as in this example, an integro-differential generalized eigenvalue problem.

2. The unstabilizing disturbances that need to be controlled in extended systems have in general a nontrivial space-time structure. Therefore a set of distributed sensors and actuators is needed to achieve a space-time controlling action.

3. The fact that, in practice, only global integrated variables are observable, restricts the feedback control to these variables only. Therefore, for extended systems, there is no guarantee that control will be achieved in general and success is only to be expected in particular favorable cases.

4. The laminar to turbulent transition, in the boundary layer, begins with the appearance of downstream moving waves which at first grow slowly and may be described by a linearized equation. After reaching a certain amplitude however, the waves develop strong three-dimensional structures

and nonlinearities and a rapid transition to turbulence becomes unavoidable. Therefore, if effective control is to be achieved, it is essential to have a fast and locally accurate feedback to tame the instabilities before the spanwise differential amplification of the TS waves begins to occur. Because it is probably very difficult to obtain the required speed and accuracy with mechanical microactuators, electromagnetic controlling schemes seem worth to explore.

5 Figure captions

Fig.1 - (a) Space stability of the spanwise modes, (b) Time stability of the spanwise modes

Fig.2 - Space instability of the streamwise scaling solution

Fig.3 - Largest Re($i\alpha_1$) for the controlled equation ($\theta_1 = 60$ and real C)

Fig.4 - Largest Re($i\alpha_1$) for the controlled equation ($\theta_1 = 60$, $|C| = 1.5$ and variable phase)

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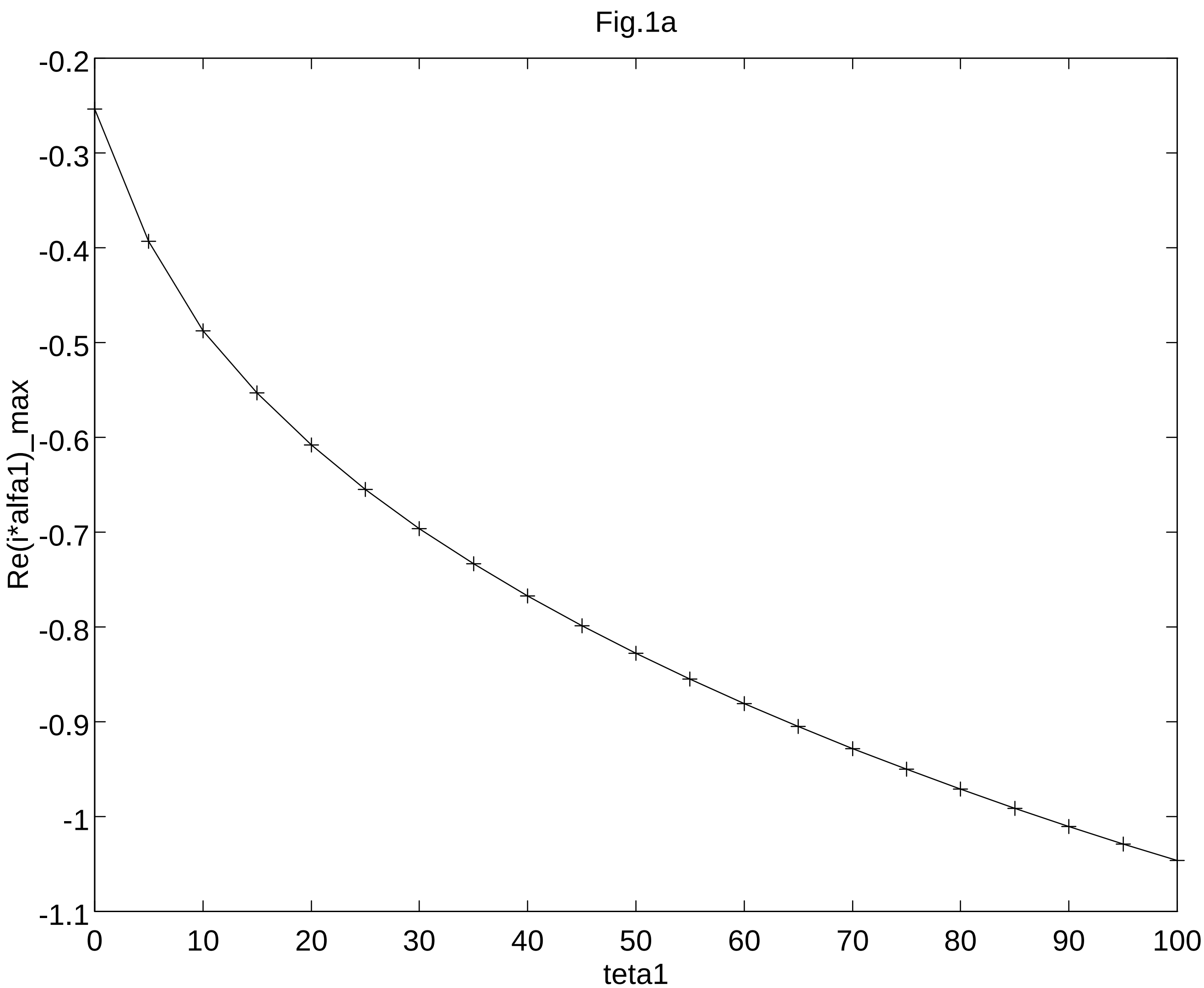

Fig.1a
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30
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teta1

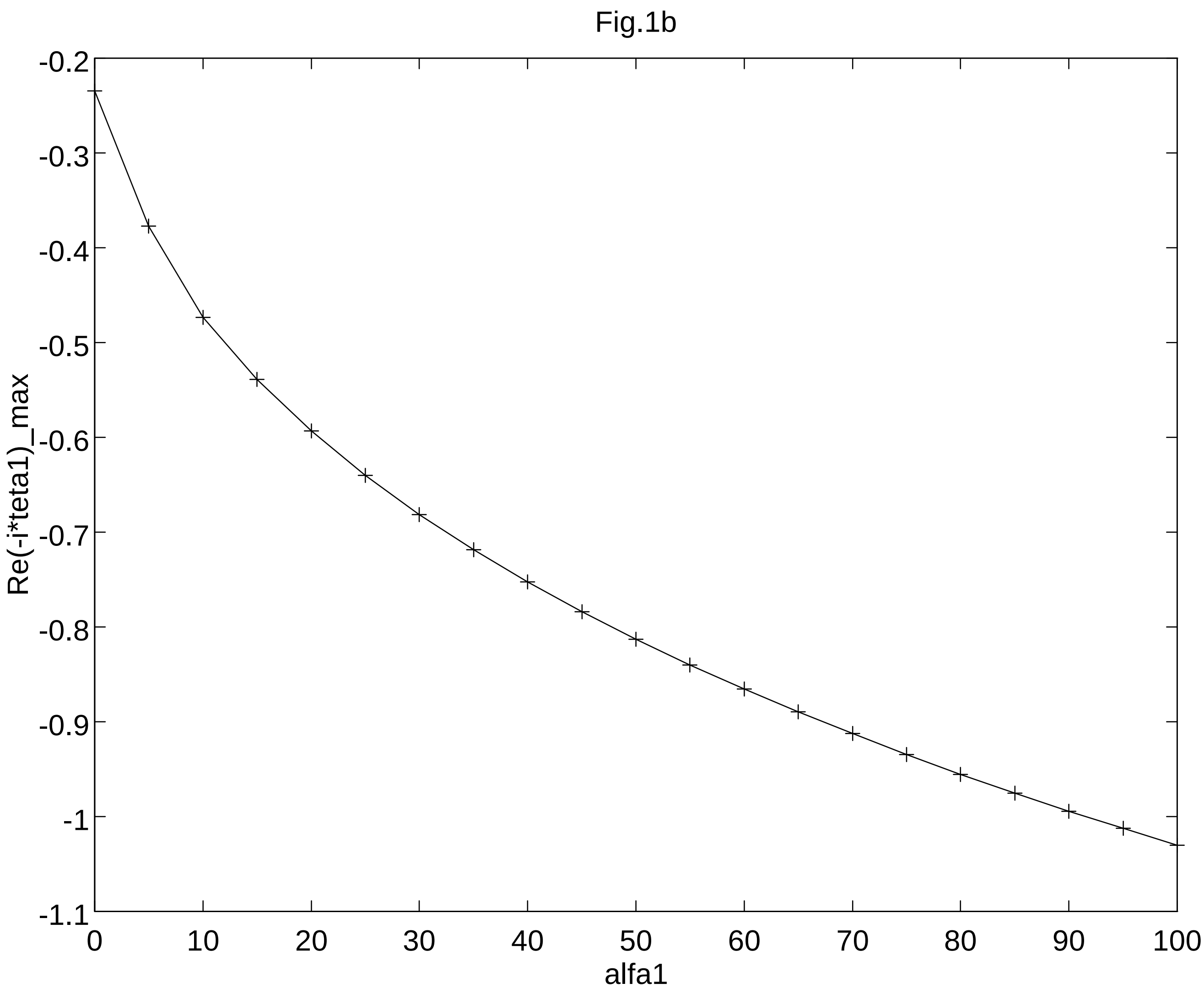
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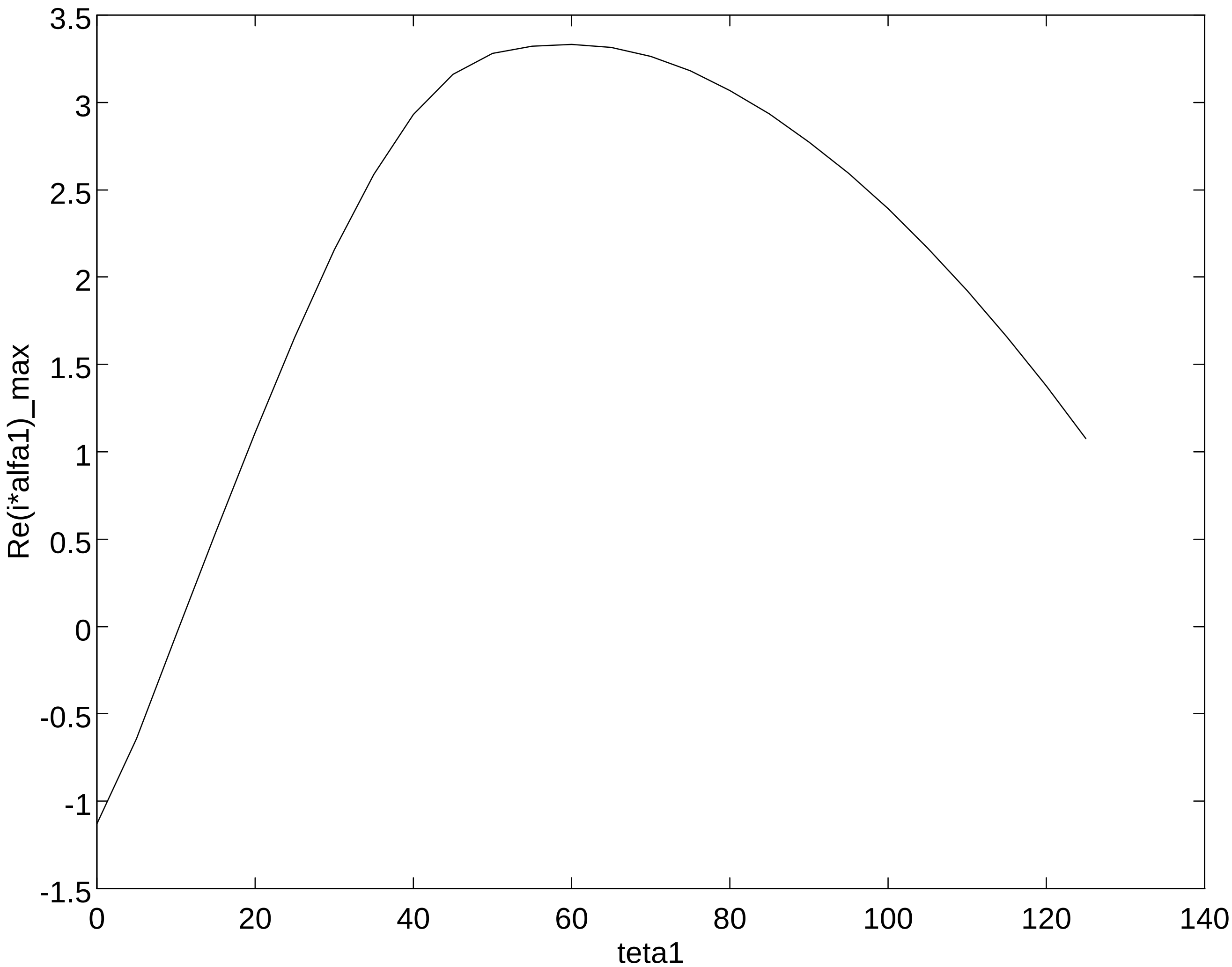
Fig.2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Re(i*alfa1)_max

Fig.3

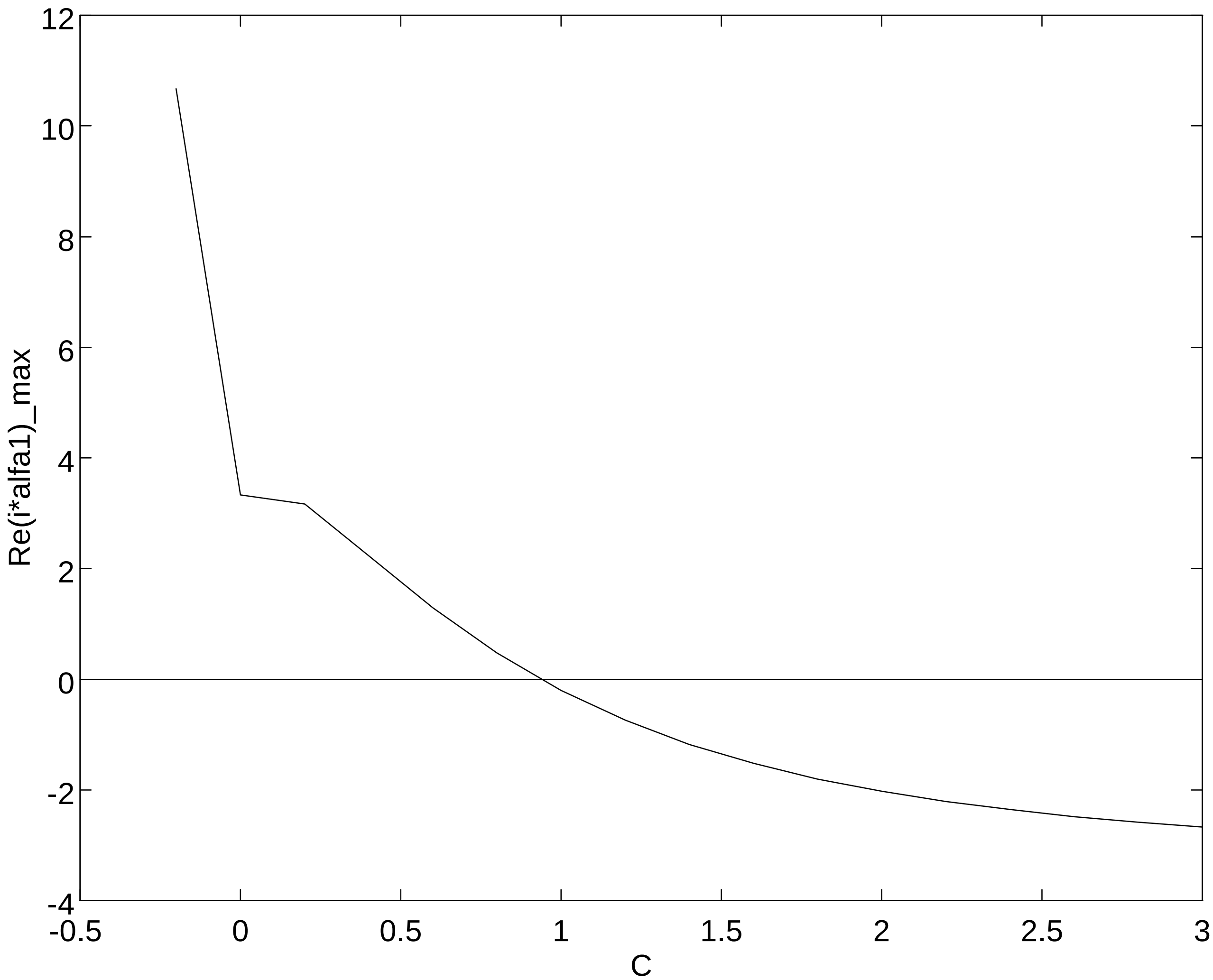

Fig.4